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NEW DIRECTION IN THE THEORY OF GRAPH INDEX IN GRAPHS

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ABSTRACT

Since 1972, several graph indices were introduced and studied. In this paper, we define the Banhatti degree of vertex in a graph. We propose the first and second E-Banhatti indices of a graph. A study of E-Banhatti indices in Mathematical Chemistry is a New Direction in the Theory of Graph Index in Graphs. Also we compute these newly defined E-Banhatti indices and their corresponding exponentials for wheel graphs, friendship graphs and some important nanostructures which are appeared in nanoscience.

Keywords: first and second E-Banhatti indices, first and second E-Banhatti polynomials, graph, nanostructure

1. INTRODUCTION

The simple, connected graph G is a graph with vertex set $V(G)$ and edge set $E(G)$. The number of vertices adjacent to the vertex u called degree of u, denoted by $d(u)$. The edge e incident by the vertices u and v with edge $uv=e$. Define $d(e) = d(u) + d(v) – 2$. For other graph terminologies and notions, the readers are referred to books [1, 2].

Mathematical Chemistry is very useful in the study of Chemical Sciences. We have found many applications in Mathematical Chemistry by using graph indices, especially in QSPR/QSAR research [3].

We define the Banhatti degree of a vertex u of a graph G as

$$B(u) = \frac{d_G(e)}{n - d_G(u)},$$

where $|V(G)|= n$ and the vertex u and edge e are incident in G.

We put forward the first and second E-Banhatti indices and these are defined as

$$EB_1(G) = \sum_{uv\in E(G)} [B(u) + B(v)],$$

$$EB_2(G) = \sum_{uv\in E(G)} B(u)B(v).$$

We propose the first and second E-Banhatti polynomials as

$$EB_1(G,x) = \sum_{uv\in E(G)} x^{B(u)+B(v)},$$

$$EB_2(G,x) = \sum_{uv\in E(G)} x^{B(u)B(v)}.$$

In Mathematical Chemstry, several graph indices were put forward and studied such as the Wiener index [4, 5, 6, 7], the Zagreb indices [8, 9, 10, 11], the Revan indices [12, 13, 14, 15], the Gourava indices [16, 17, 18, 19], the Banhatti indices [20, 21, 22, 23] and the Reverse indices [24, 25].

In this paper, we establish some usefull results on the first and second E-Banhatti indices and their corresponding polynomials.

2. SOME STANDARD GRAPHS

Proposition 1. Let G be r-regular with $|V(G)|= n$ and $r \geq 2$. Then

(i) $EB_1(G)=\frac{2nr(r-1)}{n-r}$,

(ii) $EB_2(G)=\frac{2nr(r-1)^2}{(n-r)^2}$.

Proof: Let G be r-regular with n vertices and $r \geq 2$. Then $|E(G)|= \frac{nr}{2}$. For any edge e in G,$d(e)$=2r−2. From definition, we deduce

(i) $EB_1(G)=\frac{nr}{2}\left[\frac{2r-2}{n-r}+\frac{2r-2}{n-r}\right]=\frac{2nr(r-1)}{n-r}$,

(ii) $EB_2(G)=\frac{nr}{2}\left[\frac{2r-2}{n-r}\times\frac{2r-2}{n-r}\right]=\frac{2nr(r-1)^2}{(n-r)^2}$.

Corollary 1.1. For a cycle C_n with $n \geq 3$ vertices,

(i) $EB_1(C_n)=\frac{4n}{n-2}$.

(ii) $EB_2(C_n)=\frac{4n}{(n-2)^2}$.

Corollary 1.2. For K_n with $n \geq 3$ vertices,

(i) $EB_1(K_n)=2n(n-1)(n-2)$,

(ii) $EB_2(K_n)=4n(n-1)(n-2)^2$.

Proposition 2. For a path P_n with $n \geq 3$ vertices,

(i) $EB_1(P_n)=2\left[\frac{1}{n-1}+\frac{2}{n-2}\right]+(n-3)\left[\frac{2}{n-2}+\frac{2}{n-2}\right]=\frac{4n^2-10n+4}{(n-1)(n-2)}$.

(ii) $EB_2(P_n)=2\left[\frac{1}{n-1}\times\frac{2}{n-2}\right]+(n-3)\left[\frac{2}{n-2}\times\frac{2}{n-2}\right]=\frac{4n^2-10n+4}{(n-1)(n-2)}$.

Proposition 3. For $K_{m,n}$ with $1 \leq m \leq n$ and $n \geq 2$,

(i) $EB_1(K_{m,n})=(m+n)(m+n-2)$,

(ii) $EB_2(K_{m,n})=(m+n-2)^2$.

Proof: Let $K_{m,n}$ be a complete bipartitegraph with $|V(K_{m,n})|= m + n$ and $|E(K_{m,n})|= mn$ such that $|V_1|= m$, $|V_2|= n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Then $d(e)= m + n -2$ for any edge e in $K_{m,m}$.

(i) $EB_1(K_{m,n})=\sum_{uv\in E(G)}[B(u)+B(v)]=mn\left[\frac{m+n-2}{m+n-n}+\frac{m+n-2}{m+n-m}\right]$

$=(m+n)(m+n-2)$.

(ii) $EB_2(K_{m,n}) = \sum_{uv\in E(G)} B(u)B(v) = mn\left[\frac{m+n-2}{m+n-n} \times \frac{m+n-2}{m+n-m}\right]$

$= (m+n-2)^2.$

Corollary 3.1. For $K_{n,n}$ with $n \geq 2$,

(i) $EB_1(K_{n,n}) = 4n(n-1).$

(ii) $EB_2(K_{n,n}) = 4(n-1)^2.$

Corollary 3.2. For $K_{1,n}$ with $n \geq 2$,

(i) $EB_1(K_{1,n}) = n^2 - 1.$

(ii) $EB_2(K_{1,n}) = (n-1)^2.$

3. WHEEL GRAPHS

A wheel graph W_n has $|V(W_n)|=n+1$ and $|E(W_n)|=2n$, see Figure 1.

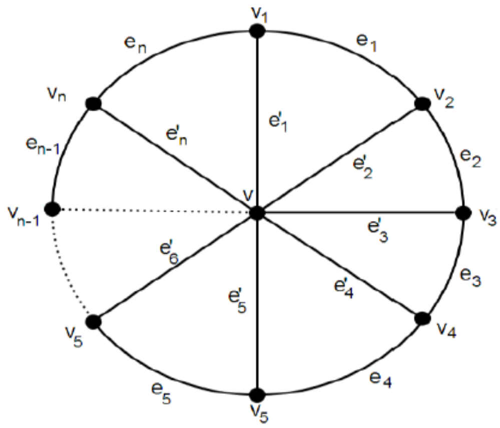

Figure 1. Wheel graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore, in W_n, we obtain that $\{B(u), B(v): uv \in E(W_n)\}$ has two Banhatti edge set partitions.

$BE_1 = \{uv \in E(W_n) \mid B(u) = B(v) = \frac{4}{n-2}\}$, $|BE_1| = n$.

$BE_2 = \{uv \in E(W_n) \mid B(u) = \frac{n+1}{n-2}, B(v) = n+1\}$, $|BE_2| = n$.

We calculate the first and second E-Banhatti indices of W_n as follows:

Theorem 1. Let W_n be a wheel graph. Then

(i) $EB_1(W_n) = \frac{n^3+7n}{n-2}.$

(ii) $EB_2(W_n) = \frac{16n}{(n-2)^2} + \frac{n(n^2+2n+1)}{(n-2)}.$

Proof: From definition and by cardinalities of the Banhatti edge partition of W_n, we obtain

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(i) $EB_1(W_n) = \sum_{uv \in E(W_n)} [B(u)+B(v)] = n\left(\frac{4}{n-2}+\frac{4}{n-2}\right)+n\left(\frac{n+1}{n-2}+n+1\right)$

By simplifying the above equation, we get the desired result.

(ii) $EB_2(W_n) = \sum_{uv \in E(W_n)} B(u)B(v) = n\left(\frac{4}{n-2}\times\frac{4}{n-2}\right)+n\left(\frac{n+1}{n-2}\times(n+1)\right)$

By solving the above equation, we get the desired result.

By using definitions and by cardinalities of the Banhatti edge partition of W_n, we obtain the first and second E-Banhatti polynomials of W_n.

Theorem 2. Let W_n be a wheel graph. Then

(i) $EB_1(W_n, x) = nx^{\frac{8}{n-2}} + nx^{\frac{n^2-1}{n-2}}$.

(ii) $EB_2(W_n, x) = nx^{\frac{16}{(n-2)^2}} + nx^{\frac{(n+1)^2}{n-2}}$.

4. FRIENDSHIP GRAPHS

The friendship graphs F_n, $n \geq 2$, have $2n+1$ vertices and $3n$ edges are shown in below graph.

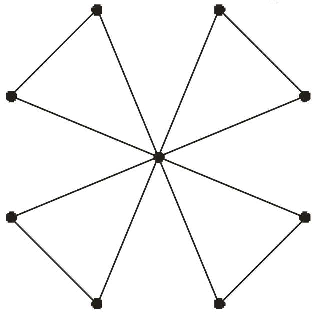

Figure 2. Friendship graph F_4

In F_n, there are two types of edges as follows:

$E_1 = \{uv \in E(F_n) \mid d(u) = d(v) = 2\}$, $|E_1| = n$.

$E_2 = \{uv \in E(F_n) \mid d(u) = 2, d(v) = 2n\}$, $|E_2| = 2n$.

Therefore, in F_n, we obtain that $\{B(u), B(v): uv \in E(W_n)\}$ has two Banhatti edge set partitions.

$BE_1 = \{uv \in E(F_n) \mid B(u) = B(v) = \frac{2}{2n-1}\}$, $|BE_1| = n$.

$BE_2 = \{uv \in E(F_n) \mid B(u) = \frac{2n}{2n-1}, B(v) = 2n\}$, $|BE_2| = 2n$.

We calculate the first and second E-Banhatti indices of F_n as follows:

Theorem 3. Let F_n be a friendship graph. Then

(i) $EB_1(F_n) = \frac{8n^3+4n}{2n-1}$.

(ii) $EB_2(F_n) = \frac{4n}{(2n-1)^2} + \frac{8n^3}{(2n-1)}$.

Proof: From definition and by cardinalities of the Banhatti edge partition of F_n, we obtain

(i) $$EB_1(F_n) = \sum_{uv \in E(F_n)} [B(u) + B(v)] = n\left(\frac{2}{2n-1} + \frac{2}{2n-1}\right) + 2n\left(\frac{2n}{2n-1} + 2n\right).$$

By simplifying the above equation, we obtain the desired result.

(ii) $$EB_2(F_n) = \sum_{uv \in E(F_n)} B(u)B(v) = n\left(\frac{2}{2n-1} \times \frac{2}{2n-1}\right) + 2n\left(\frac{2n}{2n-1} \times 2n\right).$$

By solving the above equation, we get the desired result.

By using definitions and by cardinalities of the Banhatti edge partition of F_n, we obtain the first and second E-Banhatti polynomials of F_n.

Theorem 4. Let F_n be a friendship graph. Then

(i) $$EB_1(F_n, x) = nx^{\frac{4}{2n-1}} + 2nx^{\frac{4n^2}{2n-1}}.$$

(ii) $$EB_2(F_n, x) = nx^{\frac{4}{(2n-1)^2}} + 2nx^{\frac{4n^2}{2n-1}}.$$

5. H-NAPHTALENIC NANOTUBES

We consider a family of *H*-Naphtalenic nanotubes which is denoted by *NHPX*[*m*, *n*], see Figure 3.

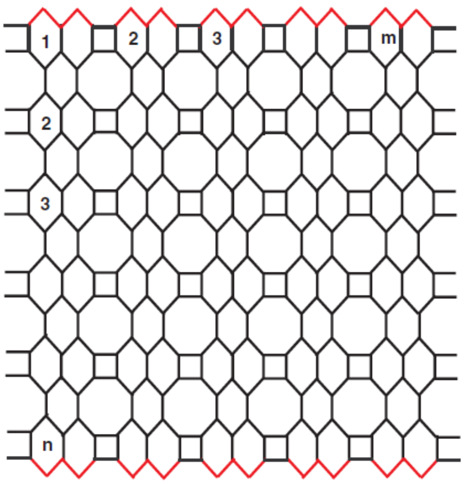

Figure 3. Graph of *H*-Naphtalenic nanotube

The graphs of a nanotube NHPX [m, n] have $10mn$ vertices and $15mn - 2m$ edges are shown in above graph. Let G= NHPX [m, n].

In G, there are two types of edges as follows:

$E_1 = \{uv \in E(G) \mid d(u) = 2, d(v) = 3\}$, $|E_1| = 8m$.

$E_2 = \{uv \in E(G) \mid d(u) = d(v) = 3\}$, $|E_2| = 15mn - 10m$.

Therefore, in $NHPX[m, n]$, we obtain that $\{B(u), B(v): uv \in E(NHPX[m, n])\}$ has two Banhatti edge set partitions.

$BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{10mn-2}, B(v) = \frac{3}{10mn-3}\}$, $|BE_1| = 8m$.

$BE_2 = \{uv \in E(G) \mid B(u) = B(v) = \frac{4}{10mn-3}\}$, $|BE_2| = 15mn-10m$.

We calculate the first and second E-Banhatti indices of *H*-Naphtalenic nanotubes as follows:

Theorem 5. Let $NHPX[m, n]$ be an *H*-Naphtalenic nanotube. Then

(i) $$EB_1(NHPX[m,n]) = \frac{120m(4mn-1)}{(10mn-2)(10mn-3)} + \frac{40(3mn-2m)}{(10mn-3)}.$$

(ii) $$EB_2(NHPX[m,n]) = \frac{72m}{(10mn-2)(10mn-3)} + \frac{80(3mn-2m)}{(10mn-3)^2}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of an *H*-Naphtalenic nanotube, we obtain

(i) $$EB_1(NHPX[m,n]) = \sum_{uv\in E(G)} [B(u)+B(v)]$$

$$= 8m\left(\frac{3}{10mn-2} + \frac{3}{10mn-3}\right) + (15mn-10m)\left(\frac{4}{10mn-3} + \frac{4}{10mn-3}\right).$$

By simplifying the above equation, we get the desired result.

(ii) $$EB_2(NHPX[m,n]) = \sum_{uv\in E(G)} B(u)B(v)$$

$$= 8m\left(\frac{3}{10mn-2} \times \frac{3}{10mn-3}\right) + (15mn-10m)\left(\frac{4}{10mn-3} \times \frac{4}{10mn-3}\right).$$

By solving the above equation, we get the desired result.

By using definitions and by cardinalities of the Banhatti edge partition of an *H*-Naphtalenic nanotube, we obtain the first and second E-Banhatti polynomials of an *H*-Naphtalenic nanotube.

Theorem 6. Let $NHPX[m, n]$ be an *H*-Naphtalenic nanotube. Then

(i) $$EB_1(NHPX[m,n],x) = 8mx^{\frac{15(4mn-1)}{(10mn-2)(10mn-3)}} + (15mn-10m)x^{\frac{8}{10mn-3}}.$$

(ii) $$EB_2(NHPX[m,n],x) = 8mx^{\frac{9}{(10mn-2)(10mn-3)}} + (15mn-10m)x^{\frac{16}{(10mn-3)^2}}.$$

6. HC_5C_7 [*p, q*] NANOTUBES

We consider $HC_5C_7[p, q]$ nanotubes, see Figure 4.

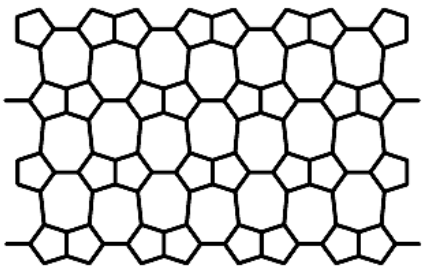

Figure 4. 2-*D* lattice of HC_5C_7 [8, 4] nanotube

The graphs of a nanotube $HC_5C_7[p, q]$ have $4pq$ vertices and $6pq - p$ edges are shown in above graph. Let $G = HC_5C_7[p, q]$.

In *G*, there are two types of edges as follows:

$E_1 = \{uv\in E(G) \mid d(u)=2, d(v) = 3\}$, $|E_1| = 4p$.

$E_2 = \{uv\in E(G) \mid d(u)= d(v) = 3\}$, $|E_2| = 6pq - 5p$.

Therefore, in G, we obtain that $\{B(u), B(v): uv\in E(NHPX[m, n])\}$has two Banhatti edge set partitions.

$BE_1 = \{uv\in E(G) \mid B(u) = \frac{3}{4pq-2}, B(v) = \frac{3}{4pq-3}\}$, $|BE_1| = 4p$.

$BE_2 = \{uv\in E(G) \mid B(u) = B(v) = \frac{4}{4pq-3}\}$, $|BE_2| = 6pq-5p$.

We calculate the first and second E-Banhatti indices of a nanotube $HC_5C_7[p,q]$ as follows:

Theorem 7. Let $HC_5C_7[p,q]$ be a nanotube. Then

(i) $$EB_1\left(HC_5C_7[p,q]\right) = \frac{12p(8pq-5)}{(4pq-2)(4pq-3)} + \frac{8(6pq-5p)}{(4pq-3)}.$$

(ii) $$EB_2\left(HC_5C_7[p,q]\right) = \frac{36p}{(4pq-2)(4pq-3)} + \frac{16(6pq-5p)}{(4pq-3)^2}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of $HC_5C_7[p,q]$, we obtain

(i) $$EB_1\left(HC_5C_7[p,q]\right) = \sum_{uv\in E(G)} [B(u)+B(v)]$$
$$= 4p\left(\frac{3}{4pq-2} + \frac{3}{4pq-3}\right) + (6pq-5p)\left(\frac{4}{4pq-3} + \frac{4}{4pq-3}\right).$$

By solving the above equation, we get the desired result.

(ii) $$EB_2\left(HC_5C_7[p,q]\right) = \sum_{uv\in E(G)} B(u)B(v)$$
$$= 4p\left(\frac{3}{4pq-2} \times \frac{3}{4pq-3}\right) + (6pq-5p)\left(\frac{4}{4pq-3} \times \frac{4}{4pq-3}\right).$$

By simplifying the above equation, we get the desired result.

By using definitions and by cardinalities of the Banhatti edge partition of a $HC_5C_7[p, q]$ nanotube, we obtain the first and second E-Banhatti polynomials of a $HC_5C_7[p, q]$ nanotube.

Theorem 8. Let $HC_5C_7[p,q]$ be a nanotube. Then

(i) $$EB_1\left(HC_5C_7[p,q],x\right) = 4px^{\frac{3(8pq-5)}{(4pq-2)(4pq-3)}} + (6pq-5p)x^{\frac{8}{4pq-3}}.$$

(ii) $$EB_2\left(HC_5C_7[p,q],x\right) = 4px^{\frac{9}{(4pq-2)(4pq-3)}} + (6pq-5p)x^{\frac{16}{(4pq-3)^2}}.$$

5. CONCLUSION .

In this study, we have defined the Banhatti degree of a vertex in a graph. We have introduced the first and second E-Banhatti indices of a graph. Furthermore, we have determined these newly defined indices for some standard graphs and certain nanotubes. This study is a new direction in Graph Indices.

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